# Prediction of thermodynamic properties of polymeric liquids using a new equation of state 

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#### Abstract

In this paper, we have used a simple equation of state (EoS) to predict the density for polymeric liquid mixtures at different temperatures, pressures, and compositions. The excess molar volumes of these mixtures have been also calculated using this equation of state. Also, we have computed isothermal compressibility. A wide comparison with experimental data has been made for each thermodynamic property. The values of statistical parameters between experimental and calculated properties show the ability of this equation of state in reproducing and predicting different thermodynamic properties for studied polymeric mixtures.


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Keywords: Equation of state; Polymeric liquids; Specific volume

## 1. Introduction

Polymeric liquids are widely used for industrial and academic purposes. Poly(ethylene glycol)s (PEGs), are frequently used in the pharmaceutical and cosmetic fields as solvents, carriers, humectants, lubricants, binders, bases, and coupling agents and also for extraction, separation, and purification of biological materials [1].

During the last decade, aqueous solutions of PEGs have received considerable attention both experimentally [2-6] and theoretically [7-10]. Aqueous solutions of a closely related polymer, poly(propylene glycol) (PPG) have also been studied. The higher hydrophobicity of PPG makes it soluble in a broad range of solvents, ranging from $n$-hexane (where it shows an upper consolute temperature) to water (where it shows a lower consolute temperature) [11].

Predicting the thermodynamic properties of hydrogenbonded systems (such as PEG systems) is a challenging problem that has received considerable attention. Both activity coefficient models and equations of state (EoSs) have been developed for hydrogen-bonded fluids [12]. Different

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theoretical EoSs have been tested against extensive pressure-volume-temperature (PVT) data of pure polymers [13,14], polymer-solvent, and blend systems [15-17].

The purpose of this study is to report the results of simultaneous calculations of volumetric and thermodynamic properties such as molar density, $\rho_{\mathrm{m}}$, specific volume, $V$, excess volume, $V^{\mathrm{E}}$, and isothermal compressibility, $\kappa_{\mathrm{T}}$, for polymer melts, solutions, and blends using a new EoS derived by Goharshadi, Morsali, and Abbaspour 'the GMA EoS' [18]. A wide comparison with experimental data was made. The accuracy of the equation of state in prediction of thermodynamic properties of polymer melts and mixtures has been determined by the statistical parameters and compared with other ones calculated by the the Flory-Orwoll-Vrij (FOV), the Schotte, and the Ihm-Song-Mason (ISM) equations of state.

### 1.1. Theoretical basis

A general EoS for liquids has been recently derived by Goharshadi et al. (the GMA EoS) [18], which has been found to be valid for polar, nonpolar, and hydrogen-bonded pure fluids and non-polymeric liquid mixtures [18-21]. The equation of state is based on the average potential energy and that is
$(2 Z-1) V_{\mathrm{m}}^{3}=A(T)+B(T) \rho_{\mathrm{m}}$
where $Z, V_{\mathrm{m}}$, and $\rho_{\mathrm{m}}$ are compressibility factor, molar volume, and molar density, respectively. The intercept and the slope of this equation both depend on temperature via the equations
$A(T)=A_{0}-\frac{2 A_{1}}{R T}+\frac{2 A_{2} \ln T}{R}$
$B(T)=B_{0}-\frac{2 B_{1}}{R T}+\frac{2 B_{2} \ln T}{R}$
where $A_{0}-A_{2}$ and $B_{0}-B_{2}$ are constants. To use the GMA EoS for a liquid, the $A$ and $B$ parameters must be known. To find these parameters, one may plot $(2 Z-1) V_{\mathrm{m}}^{3}$ against $\rho_{\mathrm{m}}$ for different isotherms. The slope and intercept of the straight lines can be fitted to Eqs. (2) and (3) and from which $A_{0}$ to $A_{2}$ and $B_{0}$ to $B_{2}$ can be found, respectively.

The functions used for calculating density and isothermal compressibility using Eq. (1) are given as Eqs. (4) and (5), respectively. The constants $A_{0}-A_{2}$ and $B_{0}-B_{2}$ can be used to calculate these properties of liquid polymer systems at any temperature and pressure.

$$
\begin{align*}
& \left(\frac{B_{0} R T}{2}-B_{1}+T B_{2} \ln T\right) \rho_{\mathrm{m}}^{5}+\left(\frac{A_{0} R T}{2}-A_{1}+A_{2} T \ln T\right) \rho_{\mathrm{m}}^{4} \\
& \quad+\frac{\rho_{\mathrm{m}} R T}{2}-P=0 \tag{4}
\end{align*}
$$

$\kappa_{T}=\frac{2}{\rho_{\mathrm{m}} R T+4 \rho_{\mathrm{m}}^{4}\left(R T A_{0}-2 A_{1}+2 T A_{2} \ln T\right)+5 \rho_{m}^{5}\left(B_{0} R T-2 B_{1}+2 B_{2} T \ln T\right)}$
values of the constants of Eqs. (2) and (3). The ranges of pressure and temperature of these tables are the same as Tables 1 and 2.

A more sensitive test for the EoS is to calculate density at different temperatures, pressures, and compositions and compare with the corresponding experimental data. Hence, the density of the binary and ternary polymer mixtures and their pure components in the wide ranges of temperature, pressure, and composition has been calculated using Eq. (4) and the results have been changed to specific volumes (in $\mathrm{cm}^{3} \mathrm{~g}^{-1}$ ) via this equation
$V=\frac{1000}{\rho_{\mathrm{m}} M_{\mathrm{w}}}$
which $M_{\mathrm{w}}$ is the molecular weight of the compound.
The accuracy in prediction of specific volume can be described by the average absolute deviation, AAD. The AAD characterizes that the experimental points are more or less close to calculated values. Tables 5-7 show the values of AAD of calculated specific volumes for the pure polymer melts and solvents, the binary solutions and blends, and the ternary mixtures, respectively. The ranges of pressure and temperature of these tables are the same as Tables 1 and 2.

To assess and compare the performance of the GMA EoS in prediction of specific volume of these polymeric mixtures and their pure components, the corresponding values of AAD obtained by the Flory-Orwoll-Vrij (FOV) [29], the Schotte [30], and the Ihm-Song-Mason (ISM) [31,32] EoSs have been calculated. The lower values of AAD obtained by the GMA EoS for binary and ternary mixtures support the ability of this EoS in prediction of specific volume of these liquid systems.

Table 1
The intercept (A), slope (B), the square of correlation coefficient (R2) of Eq. (1), and pressure range of the data for binary polymeric mixtures


Table 1 (continued)

| $\mathrm{X}_{1}$ | T (K) | $\begin{aligned} & \mathrm{B} \\ & \left(\mathrm{~L}^{4} \mathrm{~mol}^{-4}\right) \end{aligned}$ | $\begin{aligned} & \mathrm{A} \\ & \left(\mathrm{~L}^{3} \mathrm{~mol}^{-3}\right) \end{aligned}$ | $\mathrm{R}^{2}$ | $\Delta \mathrm{P}$ <br> (MPa) | $\mathrm{X}_{1}$ | T <br> (K) | $\begin{aligned} & \mathrm{B} \\ & \left(\mathrm{~L}^{4} \mathrm{~mol}^{-4}\right) \end{aligned}$ | $\begin{aligned} & \mathrm{A} \\ & \left(\mathrm{~L}^{3} \mathrm{~mol}^{-3}\right) \end{aligned}$ | $\mathrm{R}^{2}$ | $\Delta \mathrm{P}$ <br> (MPa) |
| :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: |
| 0.13477 | 298.15 | 9.45101 | -25.6901 | 0.999753 | 0.1-40 | 0.37701 | 298.15 | 1.48624 | -5.88177 | 0.999986 | 0.1-40 |
|  | 308.15 | 9.04112 | -24.3849 | 0.999747 |  |  | 308.15 | 1.41916 | -5.57178 | 0.999922 |  |
|  | 318.15 | 8.72766 | -23.3555 | 0.999902 |  |  | 318.15 | 1.36649 | -5.32217 | 0.999945 |  |
|  | 328.15 | 8.3913 | -22.2781 | 0.999889 |  |  | 328.15 | 1.31313 | -5.0727 | 0.999961 |  |
| 0.61406 | 298.15 | 0.169398 | -1.04511 | 0.999839 | $0.1-40$ | 0.8 | 298.15 | 26.4777 | -58.4786 | 0.999937 | 0.1-50 |
|  | 308.15 | 0.16327 | -0.99948 | 0.999923 |  |  | 318.15 | 24.0713 | -52.3174 | 0.999868 |  |
|  | 318.15 | 0.156333 | -0.94948 | 0.999712 |  |  | 348.15 | 21.1901 | -44.9325 | 0.999867 |  |
|  | 328.15 | 0.152016 | -0.91583 | 0.999847 |  | 0.9 | 298.15 | 30.3208 | -64.765 | 0.999909 | 0.1-50 |
| 0.85537 | 298.15 | 0.0032 | -0.04515 | 0.999882 | 0.1-40 |  | 318.15 | 27.6989 | -58.2317 | 0.99987 |  |
|  | 308.15 | 0.003062 | -0.04288 | 0.999923 |  |  | 348.15 | 24.4466 | -50.1466 | 0.999853 |  |
|  | 318.15 | 0.002955 | -0.04105 | 0.99983 |  | 1.0 | 298.15 | 35.2304 | -72.8249 | 0.99995 | 0.1-50 |
|  | 328.15 | 0.002848 | -0.03925 | 0.999897 |  |  | 318.15 | 32.051 | -65.22 | 0.999888 |  |
| 1.00000 | 298.15 | $3.76 \mathrm{E} \times 06$ | -0.00021 | 0.999976 | 0.1-40 |  | 348.15 | 27.9872 | -55.5816 | 0.999758 |  |
|  | 308.15 | $3.67 \mathrm{E} \times 06$ | -0.00021 | 0.999973 |  |  | PEG-200(1) + Anisole(2) ${ }^{\text {f }}$ |  |  |  |  |
|  | 318.15 | $3.61 \mathrm{E} \times-06$ | -0.00021 | 0.999977 |  | 0.0 | 298.15 | 0.01845 | $-0.18315$ | 0.999083 | 10-50 |
|  | 328.15 | $3.61 \mathrm{E} \times 06$ | -0.00021 | 0.999966 |  |  | 318.15 | 0.017047 | -0.16661 | 0.999835 |  |
|  | PPG-42 | + PEGME-3 | 0(2) ${ }^{\text {e }}$ |  |  |  | 338.15 | 0.016734 | -0.16099 | 0.996337 |  |
| 0.0 | 298.15 | 7.55512 | -22.4199 | 0.999915 | 0.1-50 | 0.1 | 298.15 | 0.030315 | -0.25941 | 0.999543 | 10-50 |
|  | 318.15 | 6.75406 | -19.7321 | 0.999604 |  |  | 318.15 | 0.027846 | -0.23416 | 0.99998 |  |
|  | 348.15 | 6.07456 | -17.338 | 0.99993 |  |  | 338.15 | 0.028307 | -0.23406 | 0.996089 |  |
| 0.1 | 298.15 | 9.12054 | -25.8483 | 0.999983 | 0.1-50 | 0.2 | 298.15 | 0.045188 | -0.36209 | 0.997819 | 10-50 |
|  | 318.15 | 8.14531 | -22.7194 | 0.999516 |  |  | 318.15 | 0.040631 | -0.32013 | 0.999565 |  |
|  | 348.15 | 7.32677 | -19.9571 | 0.99994 |  |  | 338.15 | 0.039145 | -0.3031 | 0.998856 |  |
| 0.2 | 298.15 | 10.7522 | -29.1742 | 0.999838 | 0.1-50 | 0.3 | 298.15 | 0.066144 | -0.49876 | 0.999787 | 10-50 |
|  | 318.15 | 9.89436 | -26.4219 | 0.999702 |  |  | 318.15 | 0.0649 | -0.4822 | 0.983357 |  |
|  | 348.15 | 8.73062 | -22.7642 | 0.999963 |  |  | 338.15 | 0.05764 | -0.42084 | 0.981155 |  |
| 0.3 | 298.15 | 12.6341 | -32.9944 | 0.999986 | 0.1-50 | 0.4 | 298.15 | 0.089754 | -0.6384 | 0.999089 | 10-50 |
|  | 318.15 | 11.571 | -29.7344 | 0.999602 |  |  | 318.15 | 0.084954 | -0.59497 | 0.999975 |  |
|  | 348.15 | 10.1847 | -25.547 | 0.999917 |  |  | 338.15 | 0.075884 | -0.52273 | 0.994366 |  |
| 0.4 | 298.15 | 15.1447 | -38.142 | 0.999905 | 0.1-50 | 0.5 | 298.15 | 0.125815 | -0.84756 | 0.999294 | 10-50 |
|  | 318.15 | 13.7397 | -34.0494 | 0.999588 |  |  | 318.15 | 0.116348 | -0.77147 | 0.999878 |  |
|  | 348.15 | 12.1088 | -29.2973 | 0.999909 |  |  | 338.15 | 0.113517 | -0.74078 | 0.997355 |  |
| 0.5 | 298.15 | 17.5053 | -42.5982 | 0.999927 | 0.1-50 | 0.6 | 298.15 | 0.165757 | -1.05981 | 0.999869 | 10-50 |
|  | 318.15 | 15.7558 | -37.7339 | 0.999798 |  |  | 318.15 | 0.158156 | -0.99557 | 0.999512 |  |
|  | 348.15 | 13.9563 | -32.6288 | 0.999844 |  |  | 338.15 | 0.161343 | -1.00048 | 0.989908 |  |
| 0.6 | 298.15 | 20.0266 | -47.1506 | 0.999962 | 0.1-50 | 0.7 | 298.15 | 0.205946 | -1.25336 | 0.998873 | 10-50 |
|  | 318.15 | 18.0463 | -41.8202 | 0.999807 |  |  | 318.15 | 0.207387 | -1.24504 | 0.997922 |  |
|  | 348.15 | 15.9658 | -36.1176 | 0.999883 |  |  | 338.15 | 0.200503 | -1.18556 | 0.99929 |  |
| 0.7 | 298.15 | 23.0188 | -52.4982 | 0.999937 | 0.1-50 | 0.8 | 298.15 | 0.283548 | -1.65096 | 0.999752 | 10-50 |
|  | 318.15 | 20.8061 | -46.7016 | 0.999801 |  |  | 318.15 | 0.270585 | $-1.55283$ | 0.999956 |  |
|  | 348.15 | 18.4907 | -40.49 | 0.999819 |  |  | 338.15 | 0.259627 | -1.46861 | 0.994008 |  |
| 0.9 | 298.15 | 0.382159 | -2.12184 | 0.9993 | 10-50 | 1.0 | 298.15 | 0.473244 | -2.52853 | 0.999465 | 10-50 |
|  | 318.15 | 0.337079 | -1.84371 | 0.998882 |  |  | 318.15 | 0.454459 | -2.3966 | 0.999808 |  |
|  | 338.15 | 0.405408 | -2.19616 | 0.99675 |  |  | 338.15 | 0.421821 | -2.19282 | 0.993999 |  |

${ }^{\text {a }}$ Data sources: Lee et al. [22,23].
${ }^{\mathrm{b}}$ Data source: Lee et al. [22].
${ }^{\text {c }}$ Data source: Colin et al. [25].
${ }^{\mathrm{d}}$ Data source: Colin et al. [24].
${ }^{\mathrm{e}}$ Data source: Lee et al. [27].
${ }^{f}$ Data source: Lee et al. [26].

Isothermal compressibility, $\kappa_{T}$, and excess volume, $V^{\mathrm{E}}$, are other thermodynamic properties investigated in this study. The values of $\kappa_{T}$ have been calculated using Eq. (5) and compared with the corresponding pseudo-experimental values calculated based on the Tait equation. The results are tabulated in Table 8. The constants of the Tait equation for each mixture have been gathered from the references of Table 1.

The excess volume is related to the molecular interactions in a mixture. By definition, the excess volume of a binary system is calculated via the following equation
$V^{\mathrm{E}}=V_{\mathrm{m}}-x_{1} V_{1}^{\circ}-x_{2} V_{2}^{\circ}$
with
$V_{\mathrm{m}}=\frac{x_{1} M_{1}+x_{2} M_{2}}{\rho_{\mathrm{m}}}$

Table 2
The intercept $(A)$, slope $(B)$, the square of correlation coefficient $\left(R^{2}\right)$ of Eq. (1), and pressure range of the data for ternary polymeric mixtures of PEG-200(1)+ PEGME-350(2) + anisole (3)

| $x_{1}, x_{2}$ | $T(\mathrm{~K})$ | $B\left(\mathrm{~L}^{4} \mathrm{~mol}^{-4}\right)$ | $A\left(\mathrm{~L}^{3} \mathrm{~mol}^{-3}\right)$ | $R^{2}$ |  |
| :--- | :--- | :--- | :--- | :--- | :--- |
| $0.2,0.2$ | 298.15 | 0.286915 | -1.61872 | 0.99993 |  |
|  | 318.15 | 0.261502 | -1.45199 | 0.999488 |  |
| $0.2,0.6$ | 348.15 | 0.229972 | -1.246 | 0.999863 |  |
|  | 298.15 | 2.68966 | -9.98782 | 0.999899 |  |
| $0.6,0.2$ | 318.15 | 2.41041 | -8.81632 | 0.999686 |  |
|  | 348.15 | 1.16045 | -7.45948 | 0.999659 |  |
|  | 298.15 | 1.02757 | -5.09561 | 0.99997 |  |
| $0.35,0.325$ | 318.15 | 0.927176 | -4.44534 | 0.999519 | 0.999859 |
|  | 348.15 | 1.06229 | -3.9213 | 0.999931 |  |
|  | 298.15 | 0.952868 | -4.68953 | 0.999486 |  |
|  | 318.15 | 0.85163 | -4.14207 | 0.999886 |  |

Data source: Lee et al. [28].
Table 3
The values of constants and the square of the correlation coefficients of Eqs. (2) and (3) for binary mixtures

| $x_{1}$ | $A_{0}\left(\mathrm{~L}^{3} \mathrm{~mol}^{-3}\right)$ | $A_{1}\left(\mathrm{~L}^{4} \mathrm{~atm} \mathrm{~mol}^{-4}\right)$ | $A_{2}\left(\mathrm{~L}^{4} \mathrm{~atm} \mathrm{~mol}^{-4} \mathrm{~K}^{-1}\right)$ | $B_{0}\left(\mathrm{~L}^{4} \mathrm{~mol}^{-4}\right)$ | $B_{1}\left(\mathrm{~L}^{5} \mathrm{~atm} \mathrm{~mol}{ }^{-5}\right)$ | $B_{2}\left(\mathrm{~L}^{5} \mathrm{~atm} \mathrm{~mol}{ }^{-5} \mathrm{~K}^{-1}\right)$ |
| :---: | :---: | :---: | :---: | :---: | :---: | :---: |
| PEG-200(1) + 1-octanol(2) |  |  |  |  |  |  |
| 0.0 | 40.0619 | 89.2086 | -0.240693 | -6.35321 | -13.8946 | 0.0383143 |
| 0.1 | 16.6121 | 44.8683 | -0.0984721 | -2.6309 | -6.85289 | 0.015752 |
| 0.2 | 8.88077 | 31.5992 | -0.0512691 | -1.33471 | -4.61501 | 0.00785402 |
| 0.3 | 2.89271 | 22.3938 | -0.0143622 | -0.350488 | -3.09326 | 0.00180569 |
| 0.4 | -5.30418 | 8.0251 | 0.0354343 | 1.40596 | 0.0572098 | -0.00884611 |
| 0.5 | -28.4905 | -34.7568 | 0.17635 | 4.96699 | 6.60907 | -0.0304709 |
| 0.6 | 416.949 | 822.687 | -2.52767 | -71.6557 | -140.863 | 0.434692 |
| 0.7 | -38.8611 | -50.1707 | 0.239766 | 6.98494 | 9.73894 | -0.0427487 |
| 0.8 | 21.965 | 70.2016 | -0.128809 | -3.69248 | -11.3629 | 0.021987 |
| 0.9 | -8.7116 | 12.8522 | 0.0569343 | 1.35577 | -1.92553 | -0.00854016 |
| 1.0 | 66.6365 | 163.685 | -0.398494 | -11.655 | -27.9819 | 0.0701268 |
| PEGME-350(1) + PEG-200(2) |  |  |  |  |  |  |
| 0.1 | 142.632 | 320.913 | -0.858089 | -27.6172 | -60.9758 | 0.166809 |
| 0.2 | 187.007 | 416.714 | -1.1271 | -38.6384 | -84.5283 | 0.233806 |
| 0.3 | 234.053 | 534.219 | $-1.40462$ | -51.0183 | -114.539 | 0.30741 |
| 0.4 | -31.8625 | 48.877 | 0.215775 | 6.73468 | -10.0457 | -0.044406 |
| 0.5 | 338.045 | 782.783 | -2.02657 | -82.5888 | -187.998 | 0.497334 |
| 0.6 | 228.362 | 591.294 | -1.35887 | -57.9662 | -146.289 | 0.347656 |
| 0.7 | 406.098 | 975.055 | -2.42969 | -111.009 | -260.776 | 0.668003 |
| 0.8 | 289.673 | 770.708 | -1.72646 | -82.3444 | -212.274 | 0.495646 |
| 0.9 | 620.907 | 1465 | -3.72217 | -189.289 | -436.466 | 1.14106 |
| 1.0 | 1432.69 | 3120.81 | -8.61728 | -456.601 | -982.281 | 2.7541 |
| PEG-200(1) + PPG-400(2) |  |  |  |  |  |  |
| 0.00000 | -8.33334 | 11.611 | 0.0521067 | 1.27202 | $-1.67643$ | $-0.00752609$ |
| 0.13477 | 301.45 | 1004.11 | -1.76457 | - 102.364 | -331.065 | 0.610266 |
| 0.2839 | 266.878 | 868.003 | $-1.5445$ | -78.9297 | -256.737 | 0.462021 |
| 0.39598 | 176.611 | 549.581 | -1.04799 | -54.6745 | -160.321 | 0.33026 |
| 0.60723 | -111.344 | -92.8975 | 0.690524 | 29.7627 | 29.961 | -0.181771 |
| 0.80381 | 127.23 | 305.249 | -0.766817 | -28.339 | -65.8464 | 0.171999 |
| 0.88454 | -20.8185 | 8.7586 | 0.131931 | 4.56506 | 0.26472 | -0.0280201 |
| 1.00000 | 600.674 | 1706.55 | -3.55852 | -226.239 | -623.38 | 1.35667 |
| Water(1)+PPG(2) |  |  |  |  |  |  |
| 0.20105 | 1203.95 | 2435.72 | $-7.38051$ | -452.444 | -905.755 | 2.77156 |
| 0.37701 | 92.2481 | 277.382 | -0.543336 | -22.1775 | -64.3974 | 0.132491 |
| 0.61406 | -4.99771 | 7.34401 | 0.0327926 | 0.702584 | -0.991677 | -0.00442427 |
| 0.85537 | 0.749271 | 2.17316 | -0.00444128 | -0.0512678 | -0.142999 | 0.000308035 |
| 1.00000 | 0.0124196 | 0.0250714 | -0.000762238 | -0.00025755 | -0.000514735 | 0.00000157874 |
| PPG-425(1) + PEGME-350(2) |  |  |  |  |  |  |
| 0.0 | 1082.49 | 2516.88 | -6.47495 | -355.89 | -814.21 | 2.1379 |
| 0.1 | 1285.18 | 2976.45 | -7.68865 | -442.075 | -1007.63 | 2.65594 |
| 0.2 | -107.967 | 305.483 | 0.747222 | 59.0943 | -50.5666 | -0.377883 |
| 0.3 | 19.1816 | 634.968 | -0.00193592 | 16.4603 | -167.944 | -0.126416 |

Table 3 (continued)

| $x_{1}$ | $A_{0}\left(\mathrm{~L}^{3} \mathrm{~mol}^{-3}\right)$ | $A_{1}\left(\mathrm{~L}^{4} \mathrm{~atm} \mathrm{~mol}^{-4}\right)$ | $A_{2}\left(\mathrm{~L}^{4} \mathrm{~atm} \mathrm{~mol}{ }^{-4} \mathrm{~K}^{-1}\right)$ | $B_{0}\left(\mathrm{~L}^{4} \mathrm{~mol}^{-4}\right)$ | $B_{1}\left(\mathrm{~L}^{5} \mathrm{~atm} \mathrm{~mol}{ }^{-5}\right)$ | $B_{2}\left(\mathrm{~L}^{5} \mathrm{~atm} \mathrm{~mol}{ }^{-5} \mathrm{~K}^{-1}\right)$ |
| :---: | :---: | :---: | :---: | :---: | :---: | :---: |
| 0.4 | 533.82 | 1747.86 | -3.08984 | -181.721 | -601 | 1.06386 |
| 0.5 | 1182.1 | 3100.28 | -6.99412 | -459.702 | -1184 | 2.73942 |
| 0.6 | 1186.21 | 3193.32 | -7.00171 | -476.384 | -1258.07 | 2.83411 |
| 0.7 | 1278.45 | 3453.31 | -7.55141 | -540.939 | - 1423.15 | 3.22334 |
| 0.8 | 570.946 | 2196.49 | -3.23953 | -222.514 | -863.811 | 1.28451 |
| 0.9 | 347.619 | 1850.36 | -1.88036 | -127.507 | -728.18 | 0.70787 |
| 1.0 | 179.26 | 1725.98 | -0.799254 | -35.7944 | -657.074 | 0.124655 |
| PEG-200(1) + anisole(2) |  |  |  |  |  |  |
| 0.0 | 16.2773 | 33.6234 | -0.0987401 | $-1.67553$ | -3.41094 | 0.0101905 |
| 0.1 | 40.2539 | 80.2112 | $-0.244522$ | -4.78131 | -9.43902 | 0.0290925 |
| 0.2 | 36.4138 | 75.947 | -0.220119 | -4.60887 | -9.4873 | 0.0279293 |
| 0.3 | -84.8153 | -155.549 | 0.515604 | 11.1956 | 20.6871 | -0.0679665 |
| 0.4 | -61.3373 | -106.788 | 0.374235 | 8.76445 | 15.4932 | $-0.0533468$ |
| 0.5 | 66.2191 | 138.431 | $-0.401462$ | -10.0034 | -20.5739 | 0.0608303 |
| 0.6 | 111.3 | 221.502 | -0.678724 | -17.844 | -35.0597 | 0.109051 |
| 0.7 | -95.4494 | - 175.452 | 0.57503 | 14.9804 | 27.9716 | -0.0899258 |
| 0.8 | 4.07458 | 26.913 | $-0.0253872$ | -0.775777 | -4.12396 | 0.00520064 |
| 0.9 | 1083.5 | 2083.72 | -6.59103 | -195.844 | -375.778 | 1.19183 |
| 1.0 | -158.637 | -271.311 | 0.964439 | 29.2541 | 50.9323 | -0.177271 |

Table 4
The values of constants and the square of the correlation coefficients of Eqs. (2) and (3) for ternary mixtures of PEG-200(1) + PEGME-350(2) + anisole(3)

| $x_{1,} x_{2}$ | $A_{0}\left(\mathrm{~L}^{3} \mathrm{~mol}^{-3}\right)$ | $A_{1}\left(\mathrm{~L}^{4} \mathrm{~atm} \mathrm{~mol}^{-4}\right)$ | $A_{2}\left(\mathrm{~L}^{4} \mathrm{~atm} \mathrm{~mol}\right.$ <br> $\left.4 \mathrm{~K}^{-1}\right)$ | $B_{0}\left(\mathrm{~L}^{4} \mathrm{~mol}^{-4}\right)$ | $B_{1}\left(\mathrm{~L}^{5} \mathrm{~atm} \mathrm{~mol}^{-5}\right)$ |
| :--- | :--- | :--- | :--- | :--- | :--- | | $B_{2}\left(\mathrm{~L}^{5} \mathrm{~atm} \mathrm{~mol}^{-}\right.$ |
| :--- |
| $0.2,0.2$ |

where $V_{\mathrm{m}}$ is the molar volume of mixture. $x_{i}, V_{i}^{\circ}$, and $M_{i}$ stand for the mole fraction, molar volume, and molecular weight, respectively, for component $i$. A comparison between experimental and calculated values of excess molar volumes
has been shown in Table 8. As an example, Fig. 2 shows the variation of excess molar volume with composition for PPG-425 (1) + PEGME-350 (2) at 0.1 MPa at different temperatures.

Table 5
Results of specific volume correlation with the equations of state for 'pure' compounds

| Compound | $\mathrm{AAD}^{\mathrm{a}}\left(\mathrm{cm}^{3} \mathrm{~g}^{-1}\right)$ |  |  | ISM EoS |
| :--- | :--- | :--- | :--- | :--- |
|  | GMA EoS | FOV EoS | Schotte EoS |  |
| PEG-200 | $0.00012^{\mathrm{b}}$ | $0.00021^{\mathrm{b}}$ | $0.00007^{\mathrm{b}}$ |  |
|  | $0.00020^{\mathrm{c}}$ | $0.00017^{\mathrm{c}}$ | $0.00012^{\mathrm{c}}$ |  |
| PPG-400 | $0.00006^{\mathrm{d}}$ |  |  |  |
| PPG-425 | 0.00004 |  | $0.01006^{\mathrm{d}}$ |  |
| PEGME-350 | 0.00014 | $0.00025^{\mathrm{e}}$ | $0.00750^{\mathrm{d}}$ |  |
|  | $0.00007^{\mathrm{b}}$ | $0.00017^{\mathrm{b}}$ | $0.00025^{\mathrm{e}}$ |  |
| Anisole | $0.00010^{\mathrm{e}}$ | $0.00021^{\mathrm{e}}$ | $0.00010^{\mathrm{b}}$ |  |
| 1-Octanol | 0.00053 | $0.00029^{\mathrm{e}}$ | $0.00032^{\mathrm{c}}$ |  |

[^1]Table 6
Results of specific volume correlation with the equations of state for polymer solutions and blends

| Mixture (1)+(2) | AAD $^{\mathrm{a}}$ |  |  |  |
| :--- | :--- | :--- | :--- | :--- |
|  | GMA | FOV | Schotte | ISM |
|  | EoS | EoS | EoS | EoS |
| PEG-200+1-octanol | 0.019 | $0.040^{\mathrm{b}}$ | $0.040^{\mathrm{b}}$ | $0.421^{\mathrm{c}}$ |
| PEG-200+PPG-400 | 0.006 | $0.146^{\mathrm{d}}$ | $0.115^{\mathrm{d}}$ | $1.169^{\mathrm{c}}$ |
| PPG-425+PEGME-350 | 0.014 | $0.264^{\mathrm{d}}$ | $0.255^{\mathrm{d}}$ |  |
| PEG-200+anisole | 0.042 | $0.150^{\mathrm{e}}$ | $0.140^{\mathrm{e}}$ |  |
| PEGME-350+PEG-200 | 0.012 | $0.020^{\mathrm{b}}$ | $0.010^{\mathrm{b}}$ |  |
| Water+PPG-400 | 0.011 |  |  |  |

${ }^{\mathrm{a}} \mathrm{AAD}=(100 / n) \sum_{i=1}^{n}\left|V_{i, \text { calc }}-V_{i, \text { exp }}\right| / V_{i, \text { exp }}$.
${ }^{\mathrm{b}}$ Data source: Lee et al. [22].
${ }^{\text {c }}$ Data source: Sabzi and Boushehri [34].
${ }^{\mathrm{d}}$ Data source: Lee et al. [27].
${ }^{\mathrm{e}}$ Data source: Lee et al. [26].

Table 7
Results of specific volume correlation with the equations of state for the ternary polymer solution, PEG-200(1) + PEGME-350(2) + anisole(3)

| $x_{1}, x_{2}$ | AAD $^{\mathrm{a}}$ |  |  |
| :--- | :--- | :--- | :--- |
|  | GMA EoS | FOV EoS $^{\mathrm{b}}$ | Schotte EoS |
| $0.2,0.6$ | 0.01 | 0.12 | 0.10 |
| $0.2,0.2$ | 0.01 | 0.13 | 0.12 |
| $0.6,0.2$ | 0.02 | 0.12 | 0.11 |
| $0.35,0.325$ | 0.02 | 0.12 | 0.13 |

${ }^{\mathrm{a}}$ AAD $=(100 / n) \sum_{i=1}^{n}\left|V_{i, \text { calc }}-V_{i, \exp }\right| / V_{i, \text { exp }}$.
${ }^{\mathrm{b}}$ Data Source: Lee et al. [28].

## 3. Conclusion

The excellent mutual agreement between calculated and experimental properties such as specific volume, isothermal compressibility, and excess volume supports the ability of the GMA EoS in predicting and reproducing the experimental thermodynamic properties of liquid mixtures.

In summary, the GMA EoS contains some important features:

Table 8
Results of excess volume and isothermal compressibility correlation with the GMA EoS for polymer solution and blends

| Mixture (1)+(2) | $\kappa_{T}$ <br> $\mathrm{AAD}^{\mathrm{a}}$ | $V^{\mathrm{E}}$ <br> $\mathrm{AAD}^{\mathrm{b}} \times 10^{2}$ <br> $\left(\mathrm{~cm}^{3} \mathrm{~mol}^{-1}\right)$ |
| :--- | :--- | :--- |
| PEG-200+1-octanol | 3.15 | 3.01 |
| PEG-200+anisole | 4.36 | 6.56 |
| PPG-425+PEGME-350 | 2.23 | 2.86 |
| PEGME-350+PEG-200 | 2.21 | 3.26 |
| PEG-200+PPG-400 | 1.29 | 2.66 |
| Water+PPG-400 | $1.36^{\mathrm{c}}$ | 3.62 |

[^2]

Fig. 2. The variation of excess molar volume with composition for PPG-425 (1) + PEGME-350(2) at 0.1 MPa at different temperatures. The solid lines show our calculated values and the symbols stand for experimental ones.

1. The form of the GMA EoS is very simple. Evaluation of its coefficients is very easy.
2. The GMA EoS can predict the thermodynamic properties of polymeric liquids of studied in any temperature, pressure, and composition.

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[^1]:    ${ }^{\mathrm{a}} \mathrm{AAD}=(1 / n) \sum_{i=1}^{n}\left|V_{i, \text { calc }}-V_{i, \exp }\right|$.
    ${ }^{\mathrm{b}}$ Data source: Lee et al. [22].
    ${ }^{\text {c }}$ Data source: Lee et al. [26].
    ${ }^{\mathrm{d}}$ Data source: Sabzi and Boushehri [33].
    ${ }^{\mathrm{e}}$ Data source: Lee et al. [27].

[^2]:    ${ }^{\text {a }} \mathrm{AAD}=(100 / n) \sum_{i=1}^{n}\left|\kappa_{i, \text { calc }}-\kappa_{i, \exp }\right| / \kappa_{i, \text { exp }}$.
    ${ }^{\mathrm{b}} \mathrm{AAD}\left(\mathrm{cm}^{3} \mathrm{~mol}^{-1}\right)=(1 / n) \sum_{i=1}^{n}\left|V_{i, \text { calc }}^{\mathrm{E}}-V_{i, \text { exp }}^{\mathrm{E}}\right|$.
    ${ }^{\text {c }}$ This value has been calculated for these compositions: $x_{1}=0.37701$, 0.61406 , and 0.85537 .

